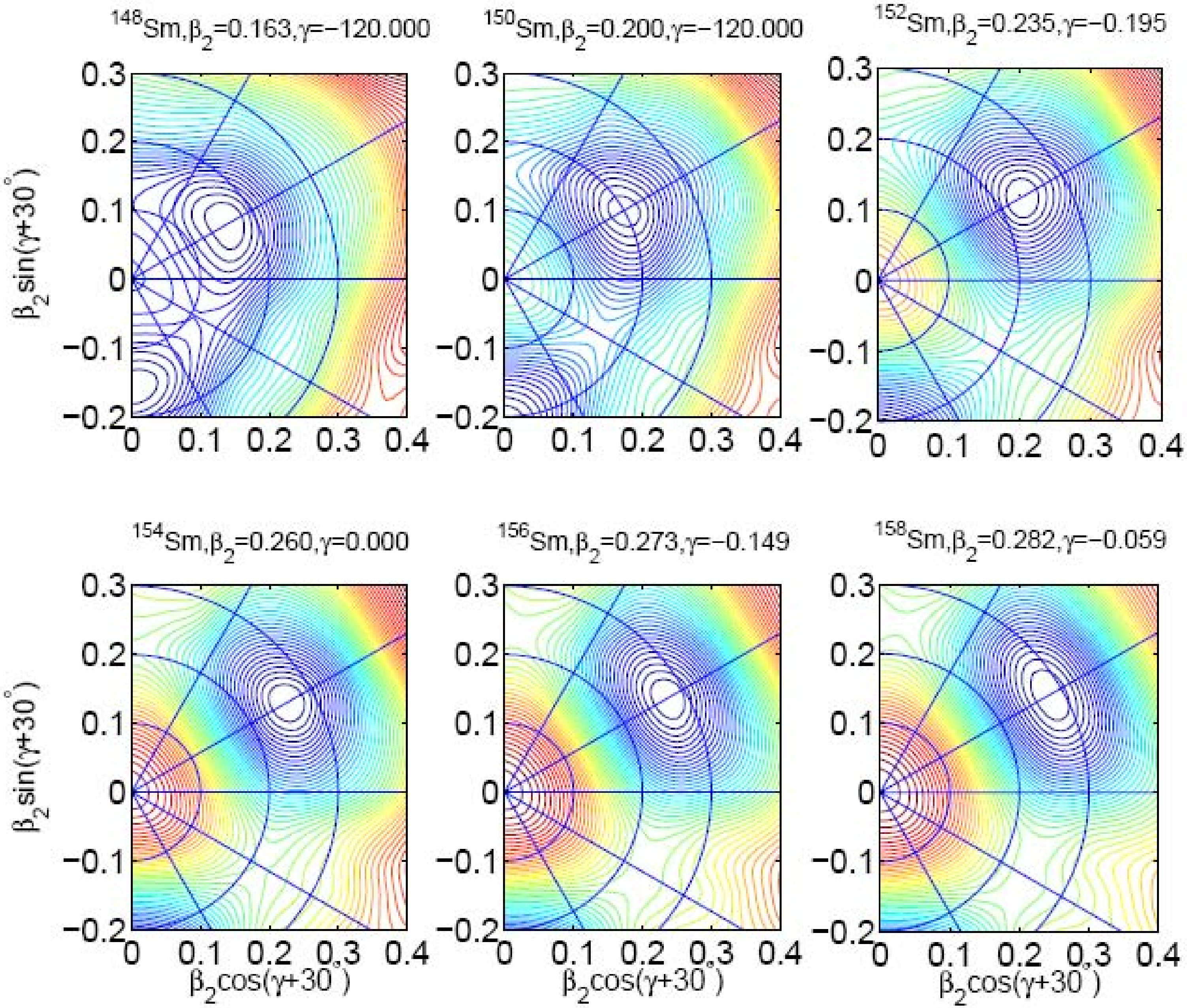
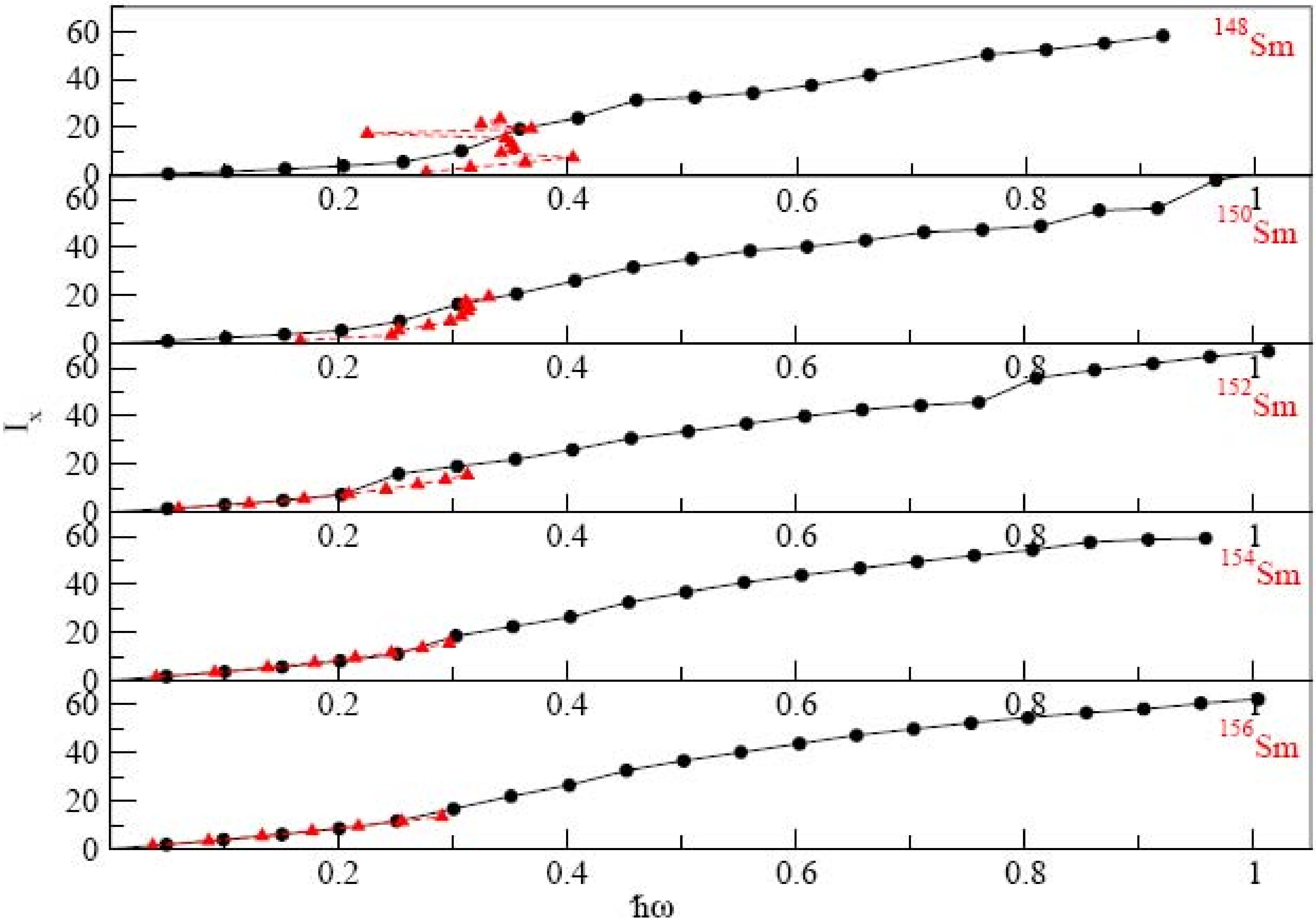
Testing spin orbit interaction at band - termination

- Spin orbit interaction crucial for evolutoin of shell gaps and drip line
- Isospin dependence of effective interactions
- Iso vector versus iso scalar potential of the spin orbit potential
- Skyrme vs RMF

Collaborators:
W. Satula, Univ Warsaw
A Baghwat, KTH
J Meng, PKU

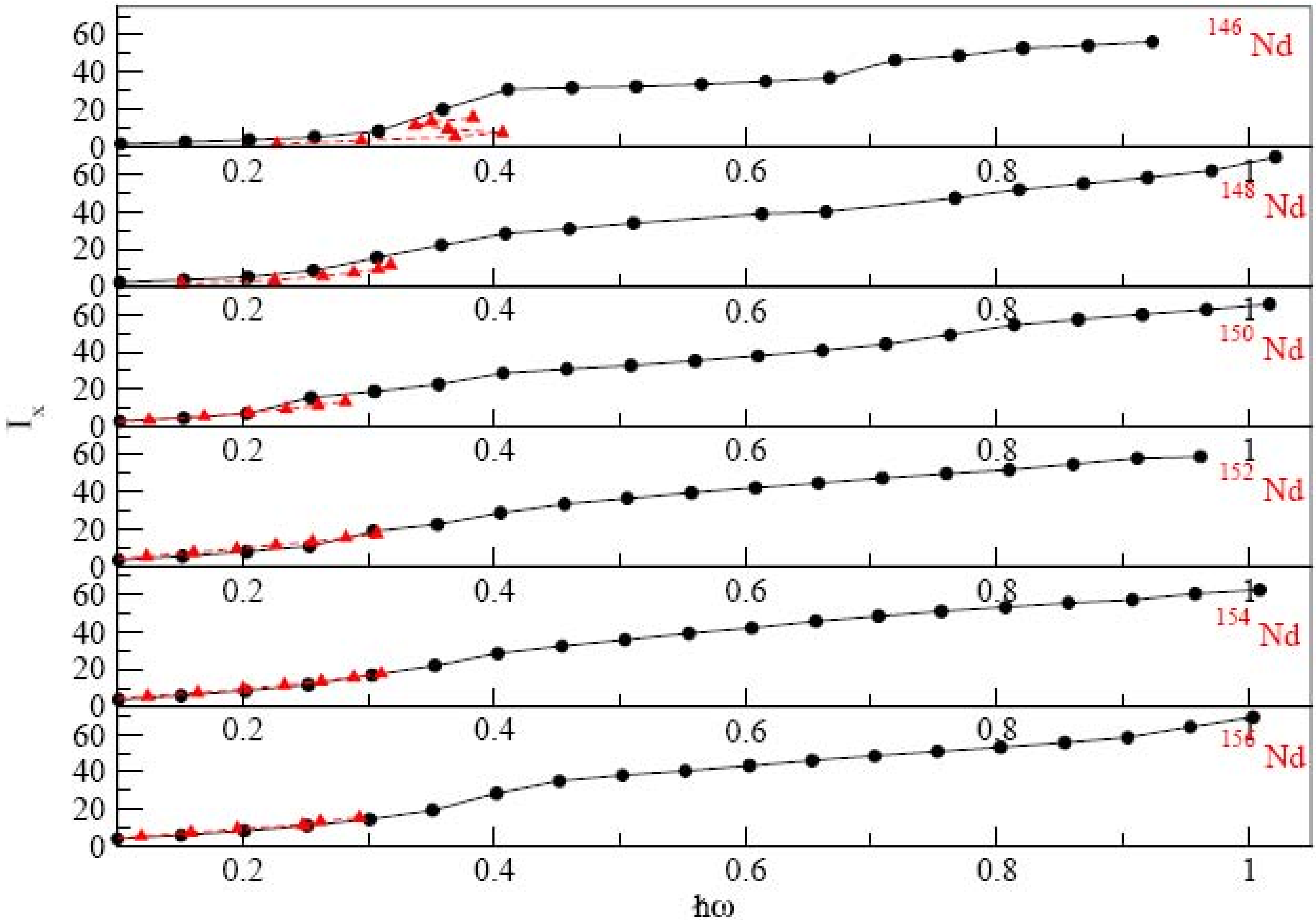
Phase transitions





^{148}Sm
^{150}Sm
^{152}Sm
^{154}Sm
^{156}Sm
I_x
ħω
0
20
40
60
0.2
0.4
0.6
0.8
1

Nd-isotopes



146 Nd
148 Nd
150 Nd
152 Nd
154 Nd
156 Nd
0
20
40
60
0.2
0.4
0.6
0.8
1
I_x
$\hbar\omega$

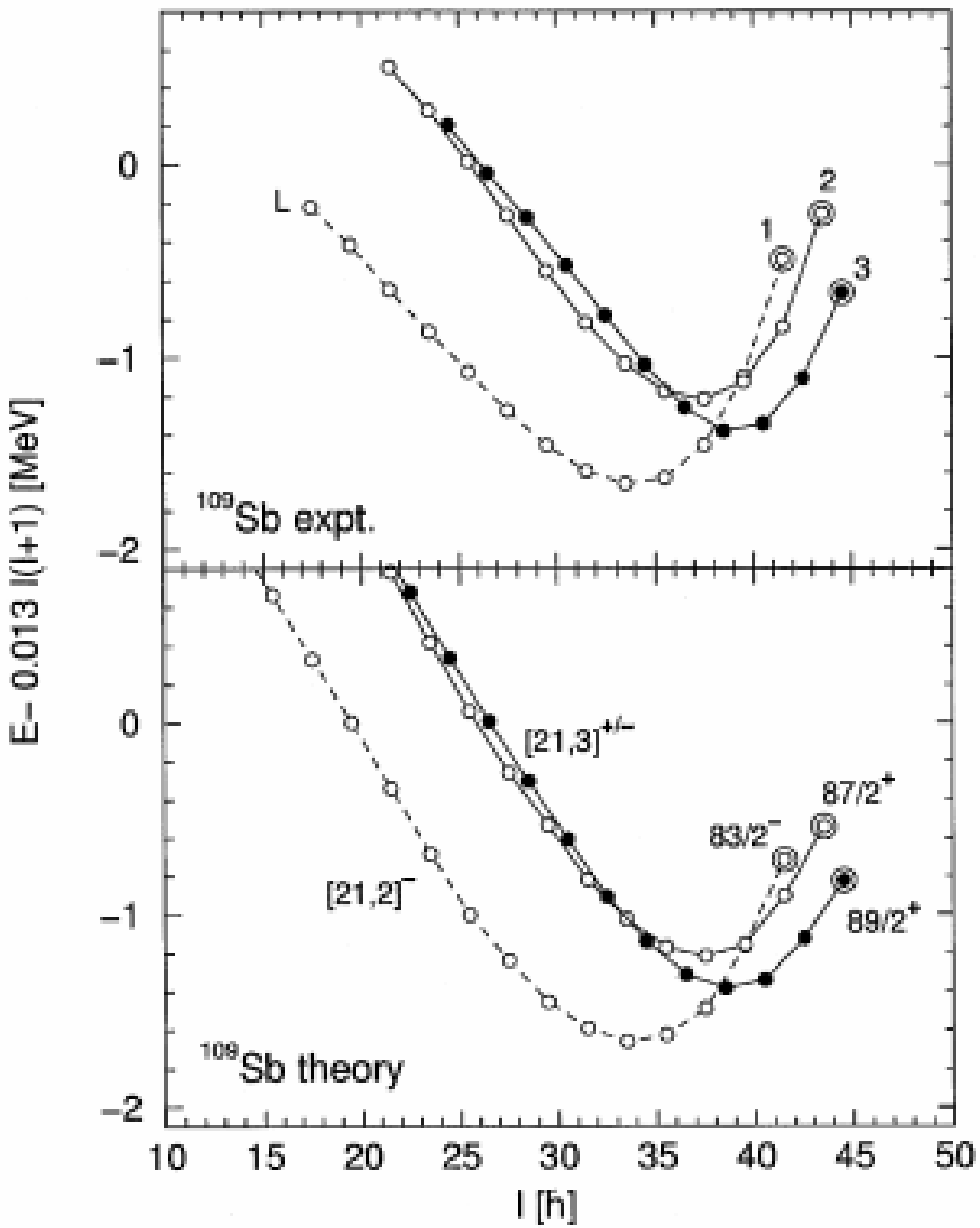
How to adjust effective forces like Skyrme, Gogny, RMF,...

- When adjusting the spin orbit force, usually s.p states are used – difficult to find ’pure’ s.p. states
- How to probe the evolution of the spin orbit field with N-Z
- time – odd components, like spin-spin fields difficult to adjust

Terminating states at high spin

- Purest s.p states available
- Deformation and pairing effects well controlled
- Sensitive to spin-spin interaction (time odd fields)
- Sensitive to spin orbit potential
- Ideal to adjust effective interactions

Nilsson Model calculations:



Select a few simple configurations:

- Particle hole excitations across the N/Z=20 shell gap
- In the Nilsson model the energy difference depends on:

28

$$\hat{H}_{Nilsson} - \frac{3}{2}\hbar\omega_o = \hbar\omega_o\left\{N - \kappa\left[2\ell s + \mu(\ell^2 - \langle\ell^2\rangle_N)\right]\right\}.$$

f7/2

20

$$\Delta e_{20} = \hbar\omega_o(1 - 6\kappa - 2\kappa\mu).$$

d3/2

e.g. 42Ca → 6^+ $(f7/2)^2$
and 11^- $(f7/2)^3$ $d3/2^{-3}$

s1/2

d5/2

Hirarchy of terms:

1. Harmonic oscilator frequency, $\hbar\omega$, governs the scale:
2. 6 κ the spin orbit strength
3. 2 $\kappa\mu$ the l^2 term

The global scale associated with $\hbar\omega$ is well adjusted via binding energies, radii etc.

The spin orbit term is not well established (fitted to selected s.p. states)

The l^2 term is not important in light nuclei (in heavy, due to Pseudo SU3 symmetry, μ=1/2)

Energy difference between f7/2^{n} and f7/2^{n+1} - d3/2^{-1}

	Ref.	$f^n_{7/2}$: $E[I_{max}]$	I_{max}	$d^{-1}_{3/2} f^{n+1}_{7/2}$: $E[I_{max}]$	I_{max}	ΔE_{exp}
$^{42}_{20}Ca_{22}$	[18]	3.189	6^+	8.297	11^-	5.108
$^{44}_{20}Ca_{24}$	[19]	10.568	8^+	5.088	13^-	5.480
$^{44}_{21}Sc_{23}$	[20]	9.141	11^+	3.567	15^-	5.574
$^{45}_{21}Sc_{24}$	[21]	5.417	$23/2^-$	11.022	$31/2^+$	5.605
	[19]			15.701	$35/2^-$	10.284
$^{45}_{22}Ti_{23}$	[19]	7.143	$27/2^-$	13.028	$33/2^+$	5.885
$^{46}_{22}Ti_{24}$	[19]	10.034	14^+	15.549	17^-	5.515
$^{47}_{23}V_{24}$	[22]	10.004	$31/2^-$	15.259	$35/2^+$	5.255

Mean exp. Energy difference ΔE=5.489, σ=0.251 (<5%)

Comments on the spin-orbit

- For nn/pp active for a pair of particles in S=1 state, T=1, L=1
- For np: either T=0,L=0 or T=1,L=1
- Uls(n)~N+Z/2=A-Z/2
- Hence: predominantly iso-vector – Opposite isovector dependence than central potential

$$\varepsilon = \frac{\Delta_{\ell s}^{(n)} - \Delta_{\ell s}^{(p)}}{(\Delta_{\ell s}^{(n)} + \Delta_{\ell s}^{(p)})/2} = \frac{2}{3}\frac{N-Z}{A}$$

$$U_{\ell s}(\tau_3) = V_{\ell s}\left(1 + \frac{1}{2}\beta_{\ell s}\frac{N-Z}{A}\cdot\tau_3\right).$$

$$\beta_{\ell s} = -2/3.$$

In Skyrme HF, the energy difference depends on time-odd spin fields and ls potential

$$\mathcal{E}^{Skyrme} = \sum_{t=0,1} \int d^3 \boldsymbol{r} \left[\mathcal{H}_t^{(TE)}(\boldsymbol{r}) + \mathcal{H}_t^{(TO)}(\boldsymbol{r}) \right].$$

$$\mathcal{H}_t^{(TE)}(\boldsymbol{r}) = C_t^{\rho} \rho_t^2 + C_t^{\Delta\rho} \rho_t \Delta \rho_t + C_t^{\tau} \rho_t \tau_t + C_t^{J} \overleftrightarrow{J}_t^{2} + C_t^{\nabla J} \rho_t \boldsymbol{\nabla} \cdot \boldsymbol{J}_t,$$

$$\mathcal{H}_t^{(TO)}(\boldsymbol{r}) = C_t^{s} \boldsymbol{s}_t^2 + C_t^{\Delta s} \boldsymbol{s}_t \Delta \boldsymbol{s}_t + C_t^{T} \boldsymbol{s}_t \cdot \boldsymbol{T}_t + C_t^{j} \boldsymbol{j}_t^2 + C_t^{\nabla j} \boldsymbol{s}_t \cdot (\boldsymbol{\nabla} \times \boldsymbol{j}_t).$$

Spin orbit in SHF

$$V_{LS}(q,\boldsymbol{r}) = -i\boldsymbol{W}_q(\boldsymbol{r})\boldsymbol{\nabla}\times\boldsymbol{\sigma}$$

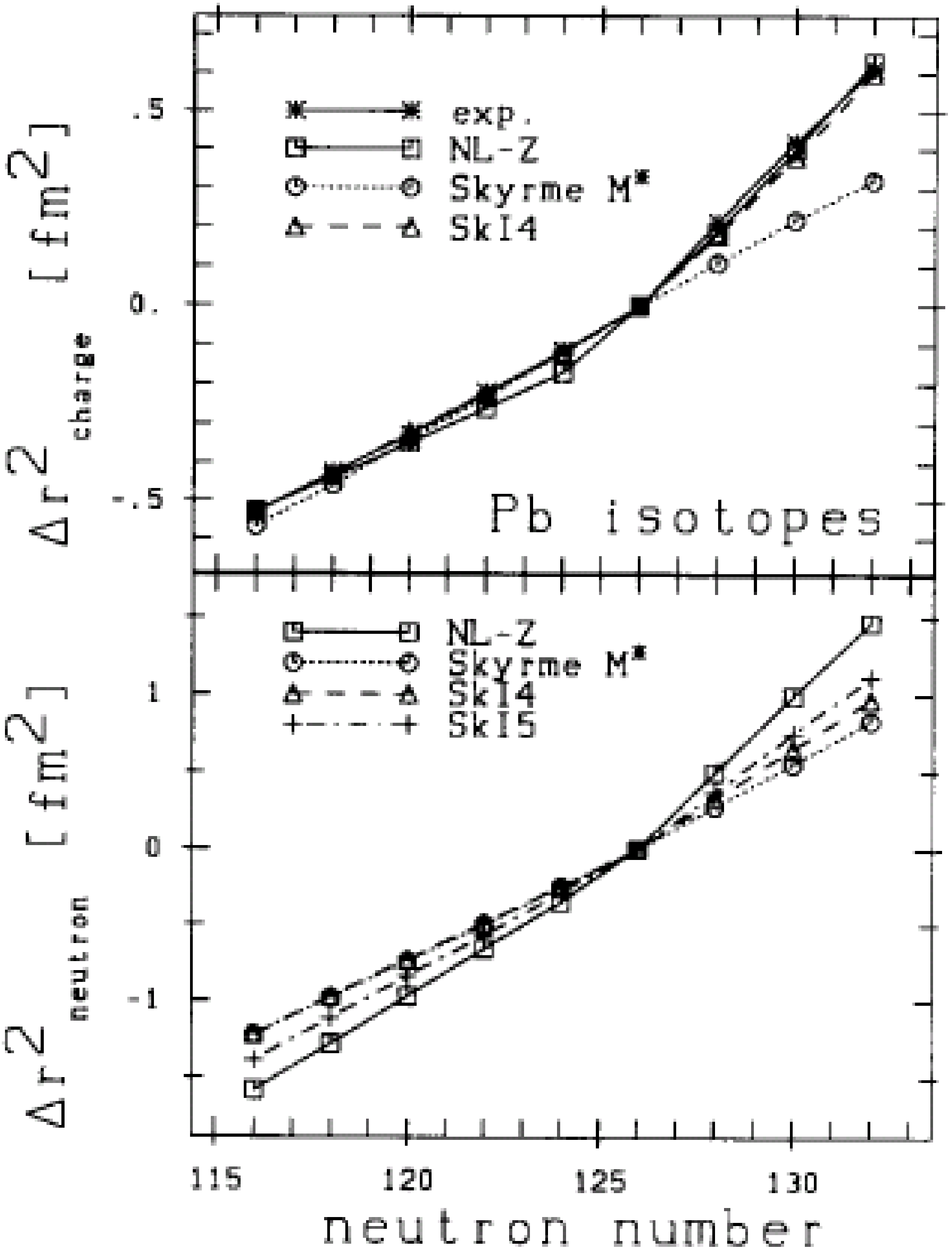
$$V_{LS}(q,r) \approx \left\{W\frac{1}{r}\rho_0'(r) + W'\frac{1}{r}\rho_q'(r)\right\}\boldsymbol{\ell s} = \left\{\frac{W_0}{r}\rho_0'(r) \pm \frac{W_1}{r}\rho_1'(r)\right\}\boldsymbol{\ell s}$$

$$\rho_0' = (\rho_n + \rho_p)' \qquad \rho_1' = (\rho_n - \rho_p)' \qquad W_0 \equiv W + \tfrac{1}{2}W' \qquad W_1 \equiv \tfrac{1}{2}W'$$

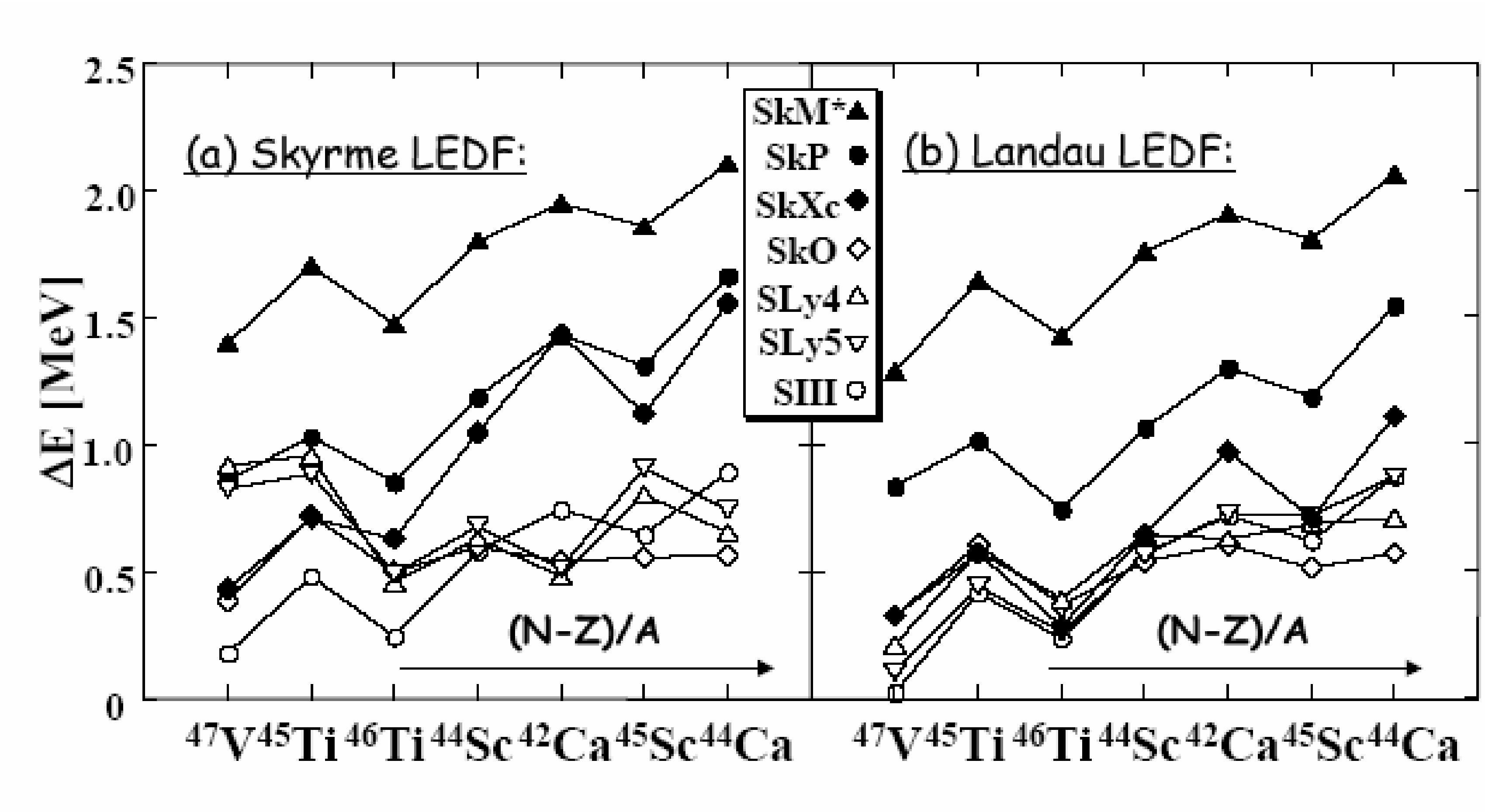
$$C_t^{\nabla J} = -\tfrac{1}{2}W_t$$

- Fock term of the Skyrme force generates strong isovector spin orbit potential

Radii in Pb-isotopes Skyrme vs RMF

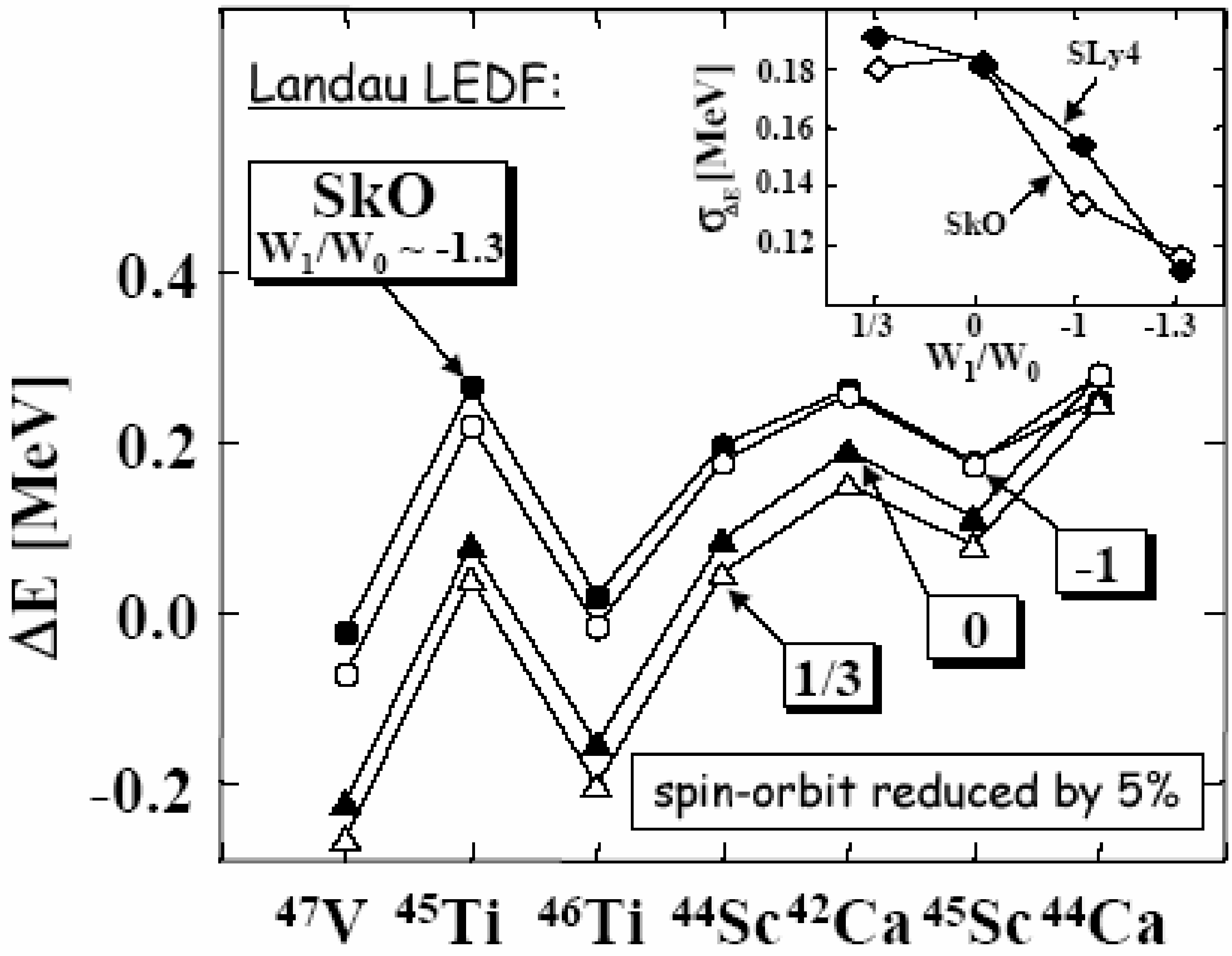


Terminating states allow for adjustment



$\Delta E = \Delta E_{exp} - \Delta E_{theo}$

Decrease spin orbit – how is the N-Z dependence?



RMF Hartree Energy

$$
\begin{aligned}
E_{\text{nucleon}} &= \sum_i \epsilon_i \ , \\
E_\sigma &= -\frac{1}{2}\int d^3r \left\{ g_\sigma \rho_s(\mathbf{r})\sigma(\mathbf{r}) + \left[\frac{1}{3}g_2\sigma(\mathbf{r})^3 + \frac{1}{2}g_3\sigma(\mathbf{r})^4\right]\right\} \\
E_\omega &= -\frac{1}{2}\int d^3r \left\{ g_\omega \rho_v(\mathbf{r})\omega^0(\mathbf{r}) - \frac{1}{2}g_4\omega^0(\mathbf{r})^4 \right\} , \\
E_\rho &= -\frac{1}{2}\int d^3r\, g_\rho \rho_3(\mathbf{r})\rho^0(\mathbf{r}), \\
E_c &= -\frac{e^2}{8\pi}\int d^3r\, \rho_c(\mathbf{r}) A^0(\mathbf{r}), \\
E_{\text{CM}} &= -\frac{3}{4}41A^{-1/3},
\end{aligned}
$$

$$
V_{tot} = V(\mathbf{r}) + \beta S(\mathbf{r}) = \beta\left[g_\omega \gamma^\mu \omega_\mu(\mathbf{r}) + g_\rho \gamma^\mu \vec{\tau}\vec{\rho}_\mu(\mathbf{r}) + g_\sigma \sigma(\mathbf{r})\right]
$$

$$
\begin{aligned}
V_{isos}(\mathbf{r}) &= \beta\left[g_\omega \gamma^\mu \omega_\mu(\mathbf{r}) + g_\sigma \sigma(\mathbf{r})\right] \\
V_{isov}(\mathbf{r}) &= \beta g_\rho \gamma^\mu \vec{\tau}\vec{\rho}_\mu(\mathbf{r})
\end{aligned}
$$

RMF Hartree Field

$$\{\alpha p + V + \beta(m - S)\}\psi_i = \epsilon_i \psi_i, \qquad V = g_\omega \omega^0$$

$$-\Delta\sigma + U'(\sigma) = -g_\sigma \rho_s, \qquad S = -g_\sigma \sigma$$

$$\{-\Delta + m_\omega^2\}\omega^0 = g_\omega \rho_v,$$

$$\begin{pmatrix} m + V - S & \sigma p \\ \sigma p & -m + V + S \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} + E \begin{pmatrix} f \\ g \end{pmatrix}.$$

$$\left\{ p \frac{1}{2\tilde{m}} p + W + \frac{1}{4\tilde{m}^2} \frac{\mathrm{d}}{\mathrm{d}r}(V + S) ls \right\} f = \varepsilon f.$$

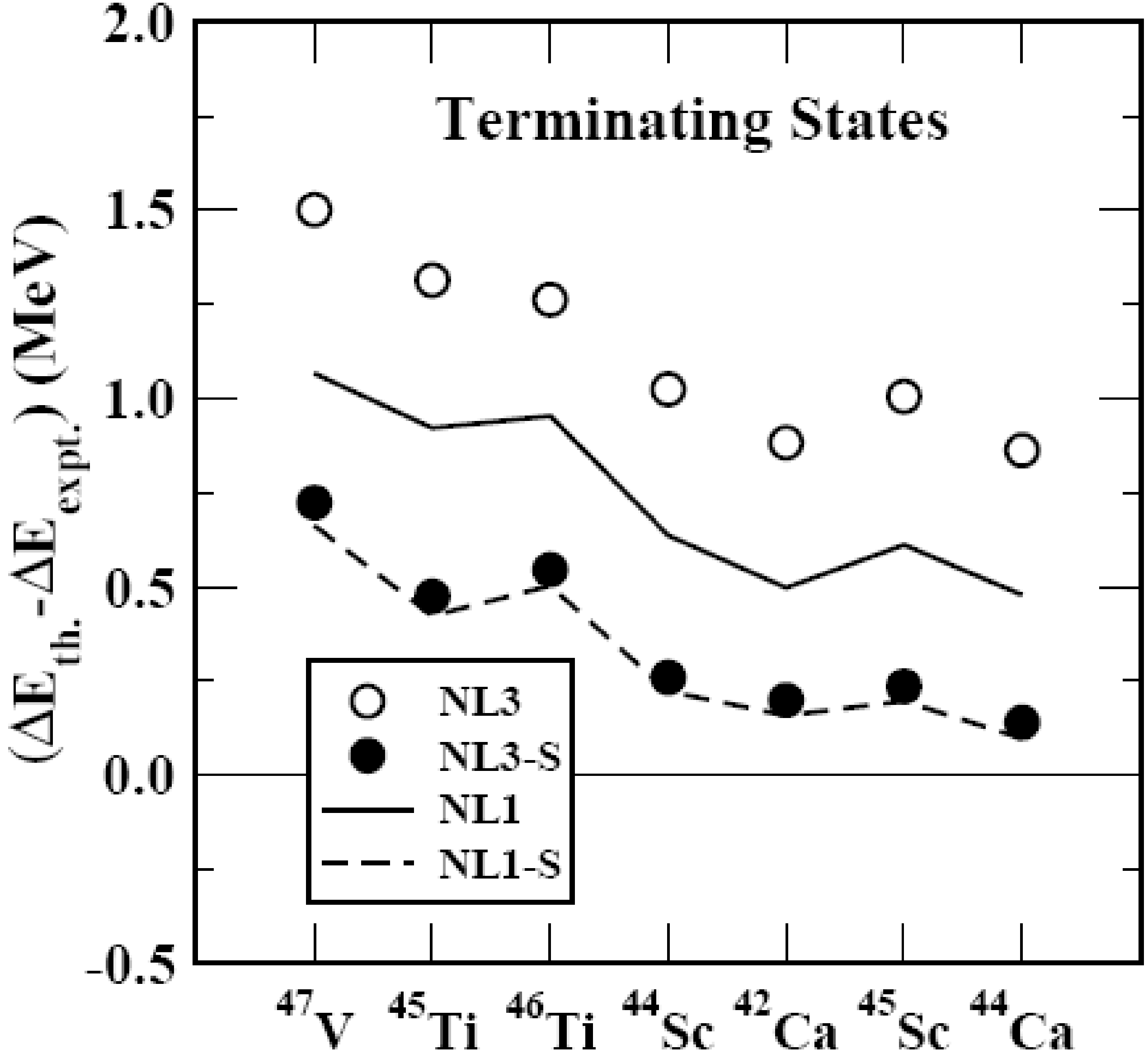
$$W = V - S \qquad \tilde{m} = m - \tfrac{1}{2}(V + S)$$

Spin orbit in RMF

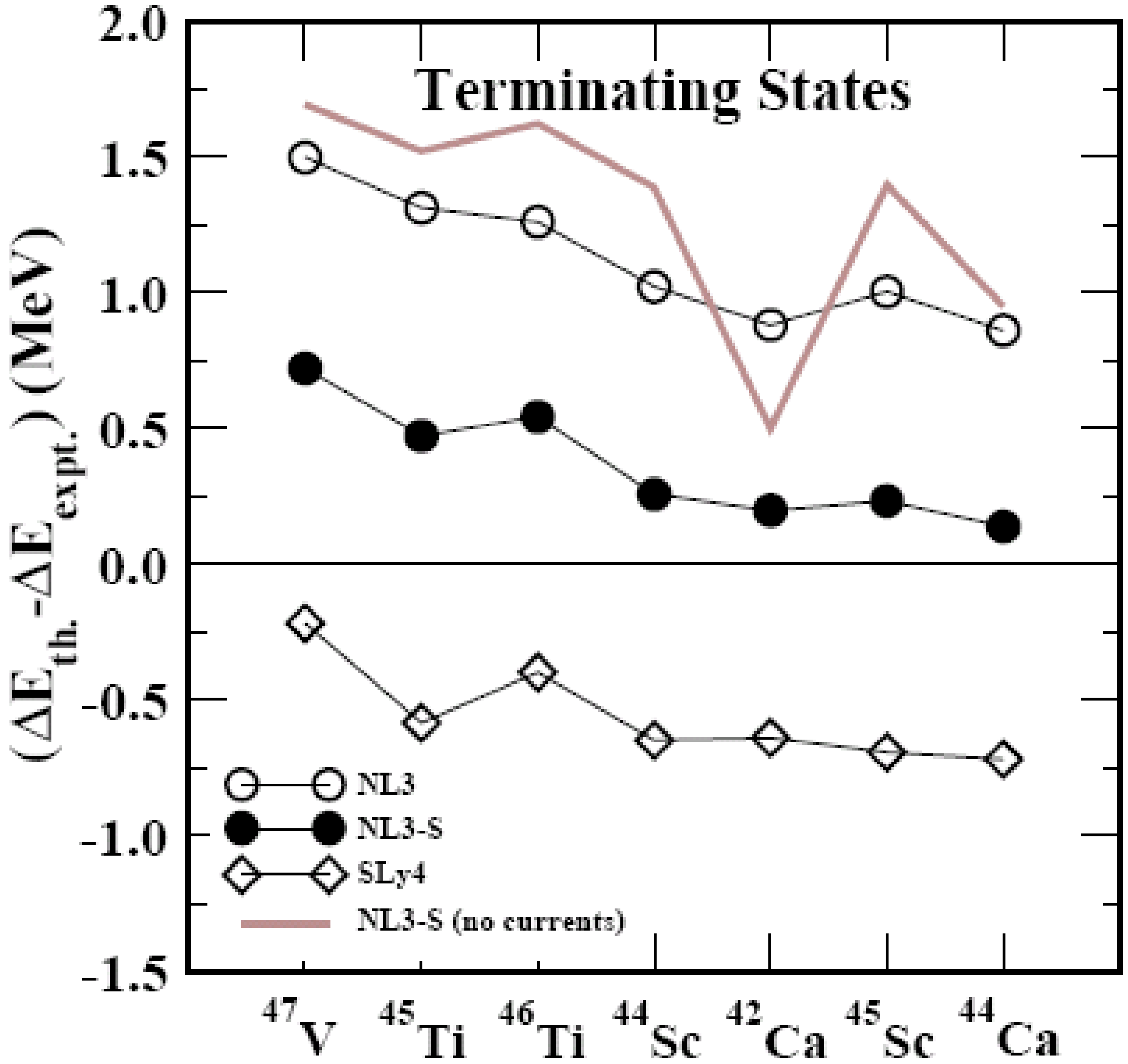
$$U_{ls}^{\tau}(\vec{r}) = \frac{1}{rm^2m^{\star 2}}\left[\left(C_\sigma^2 + C_\omega^2\right)\nabla_r\rho(\vec{r}) \pm C_\rho^2\nabla_r\rho_{pn}(\vec{r})\right]$$

$$U_{ls}^{p(n)} = \frac{-(C_\sigma^2 + C_\omega^2)A}{rm^2m^{\star 2}}\left[1 \mp \lambda\frac{N-Z}{A}\right]g(r)$$

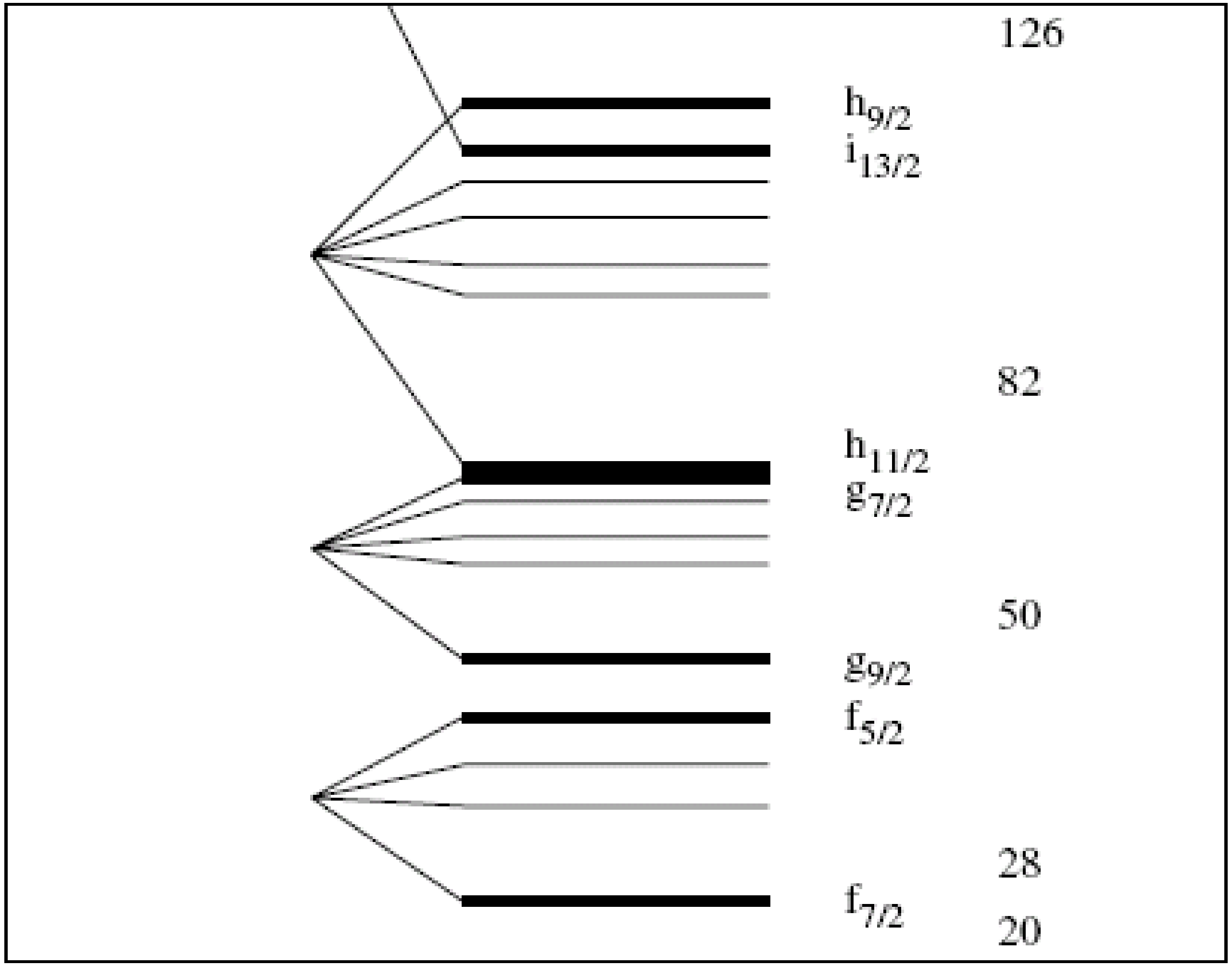
$$\lambda = C_\rho^2/(C_\sigma^2 + C_\omega^2)$$


Terminating States
$(\Delta E_{th.} - \Delta E_{expt.})$ (MeV)
2.0
1.5
1.0
0.5
0.0
-0.5
NL3
NL3-S
NL1
NL1-S
^{47}V
^{45}Ti
^{46}Ti
^{44}Sc
^{42}Ca
^{45}Sc
^{44}Ca

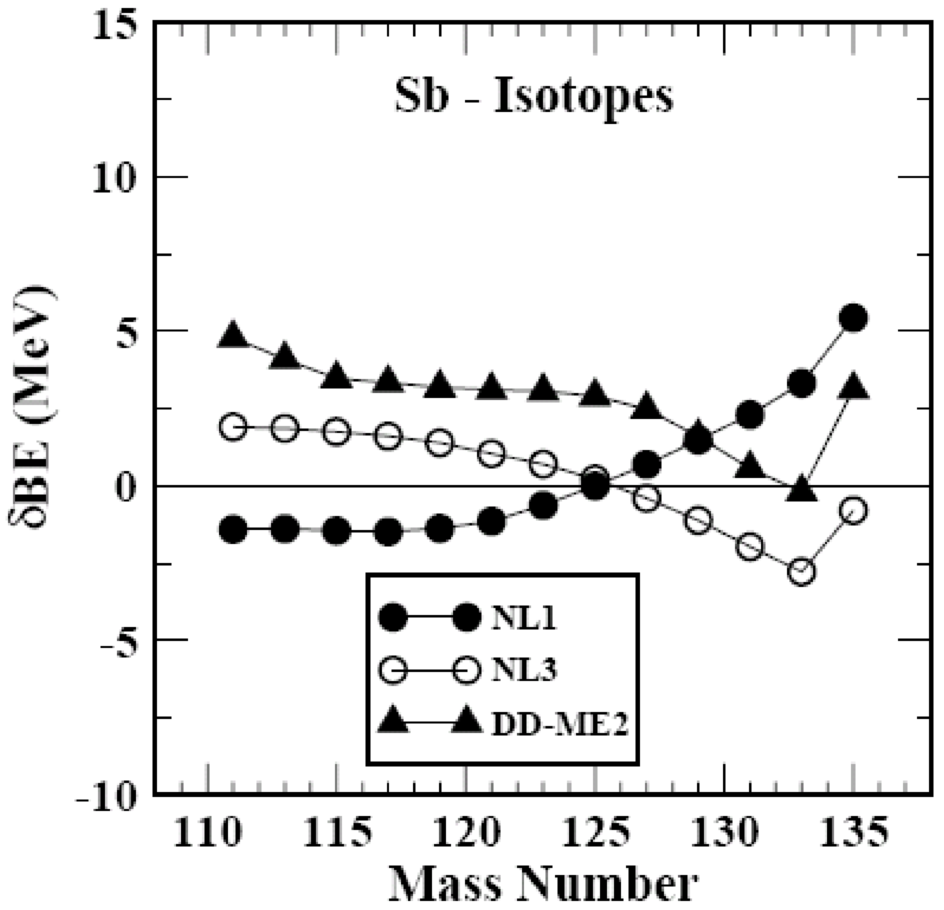
RMF vs Skyrme



Study energy difference between h11/2 and g7/2

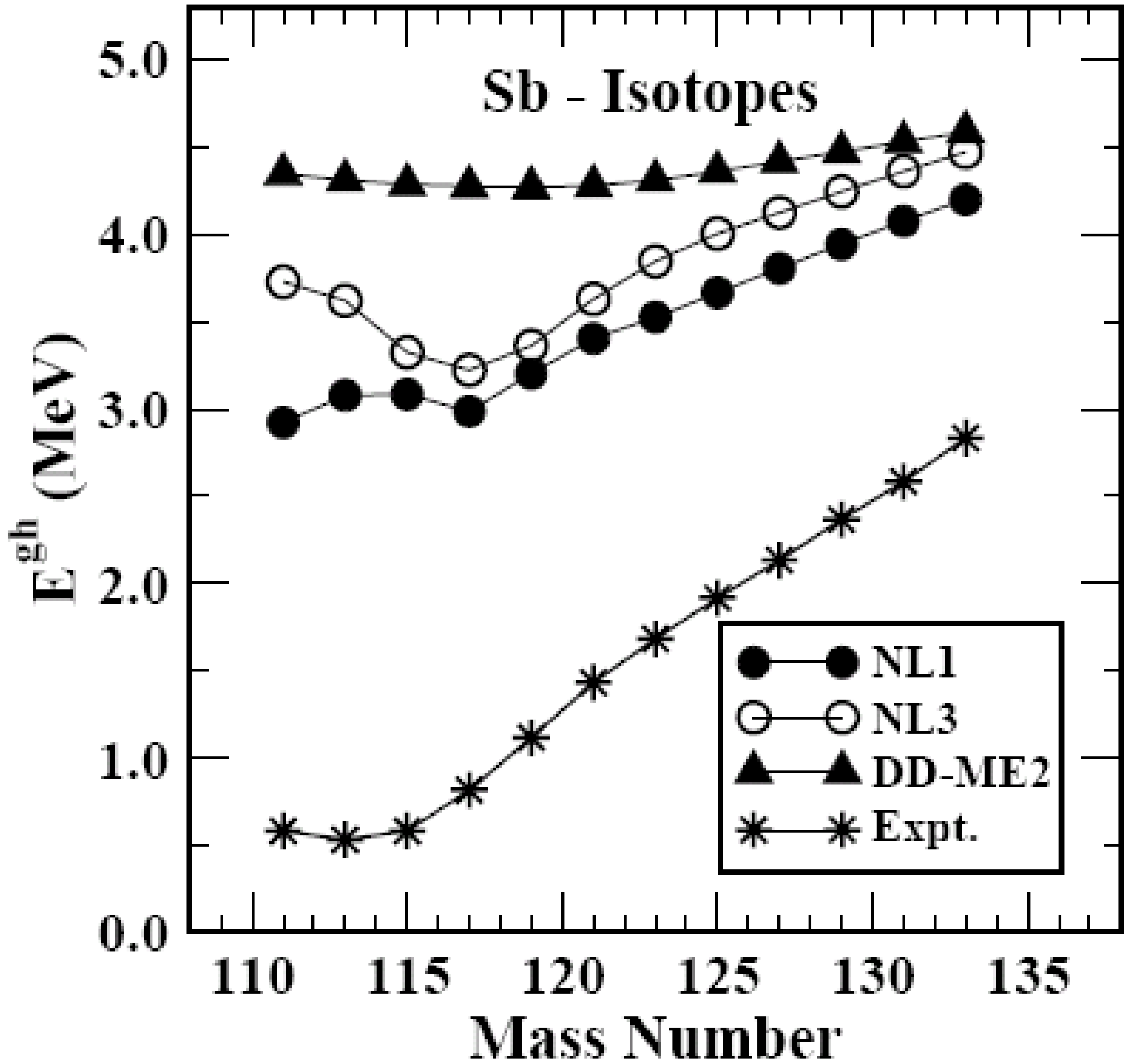


Binding energies for the Sb isotopes

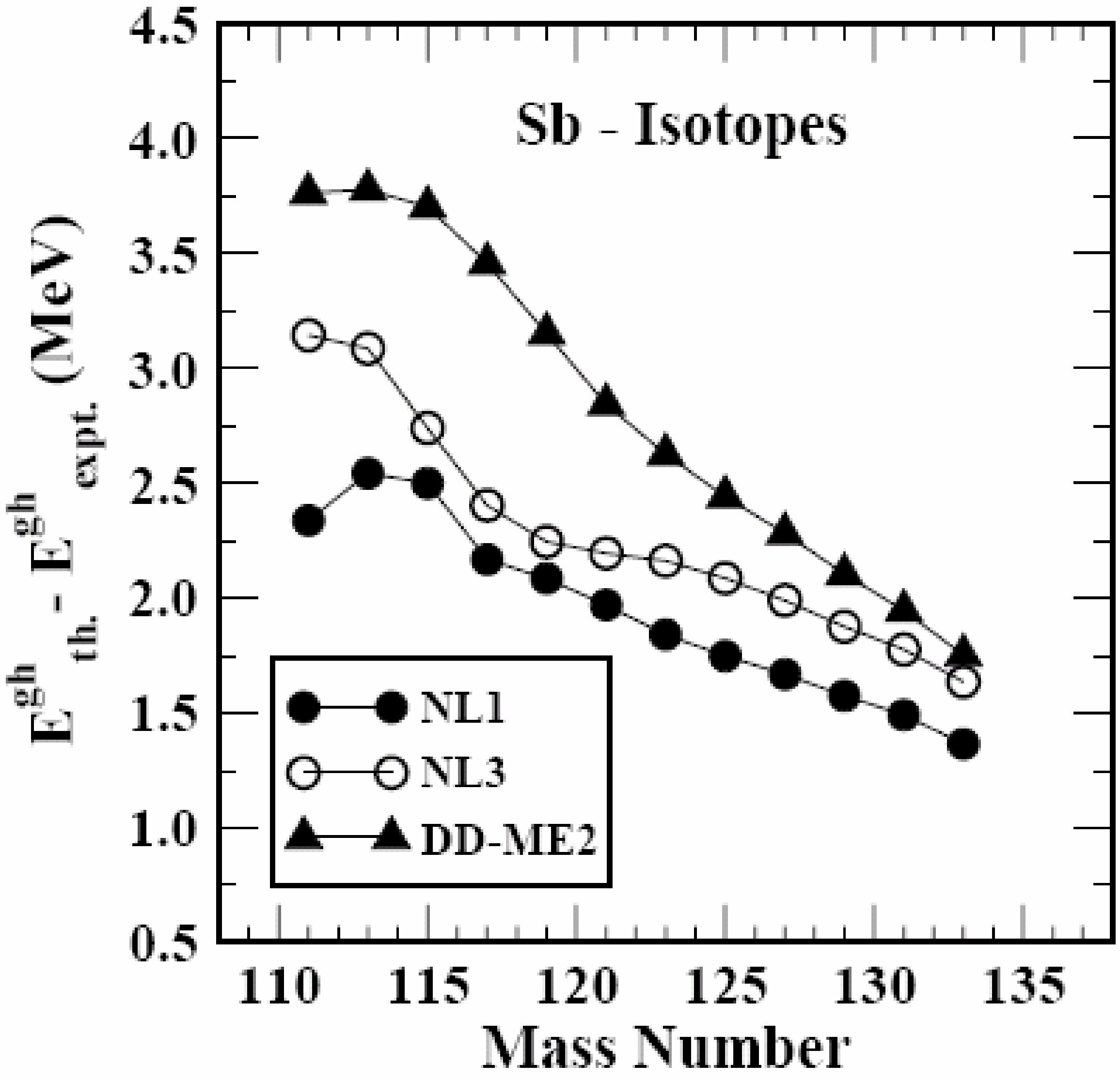


Energy difference between g7/2 and h11/2

E^{gh} = e(h11/2)-e(g7/2)



Energy difference



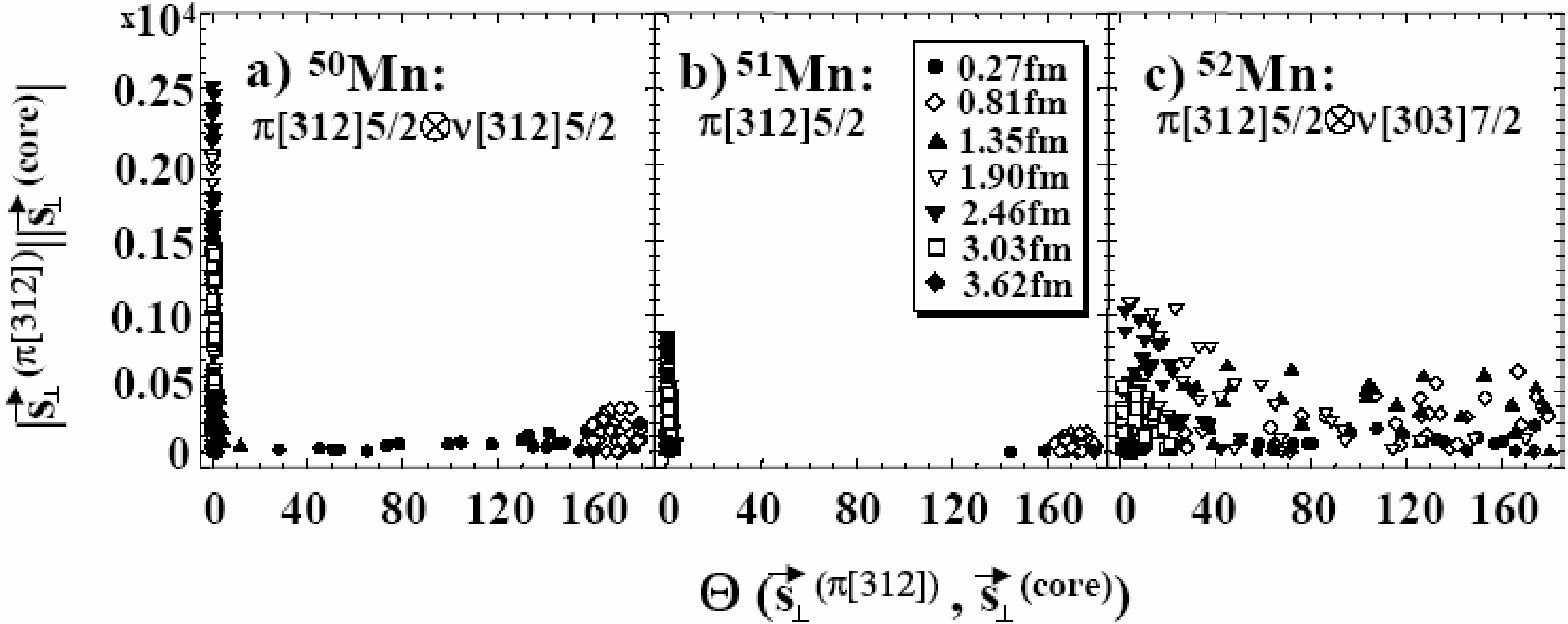
Conclusions

Band terminating states indeed excellent
tool to determine the effective interactions
Spin-spin interaction in Skyrme can be adjusted globally
Iso vector part of Spin-orbit potential in Skyrme
needs to be adjusted

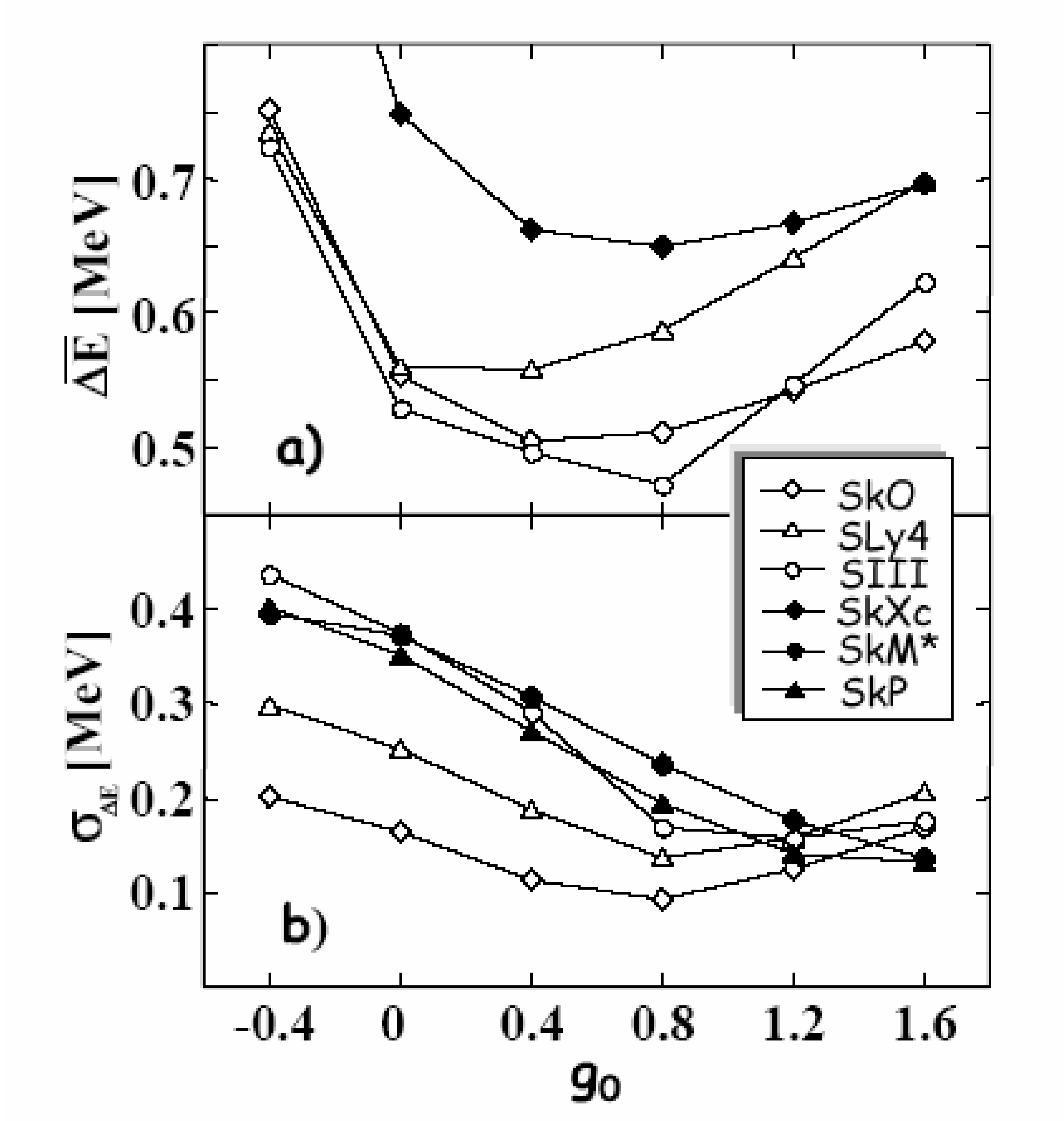
Deficencies in the effective spin orbit potential in RMF.
Apparently too few mesons to properly account for the iso-
vector dependence. (pi-meson – tensor force..?)

Wishlist – more data!

Spin-spin fields generates unphysical polarization in N=Z nuclei



Determine the Landau parameters



$$g_0 = N_0(2C_0^s + 2C_0^T \beta \rho_0^{2/3}),$$

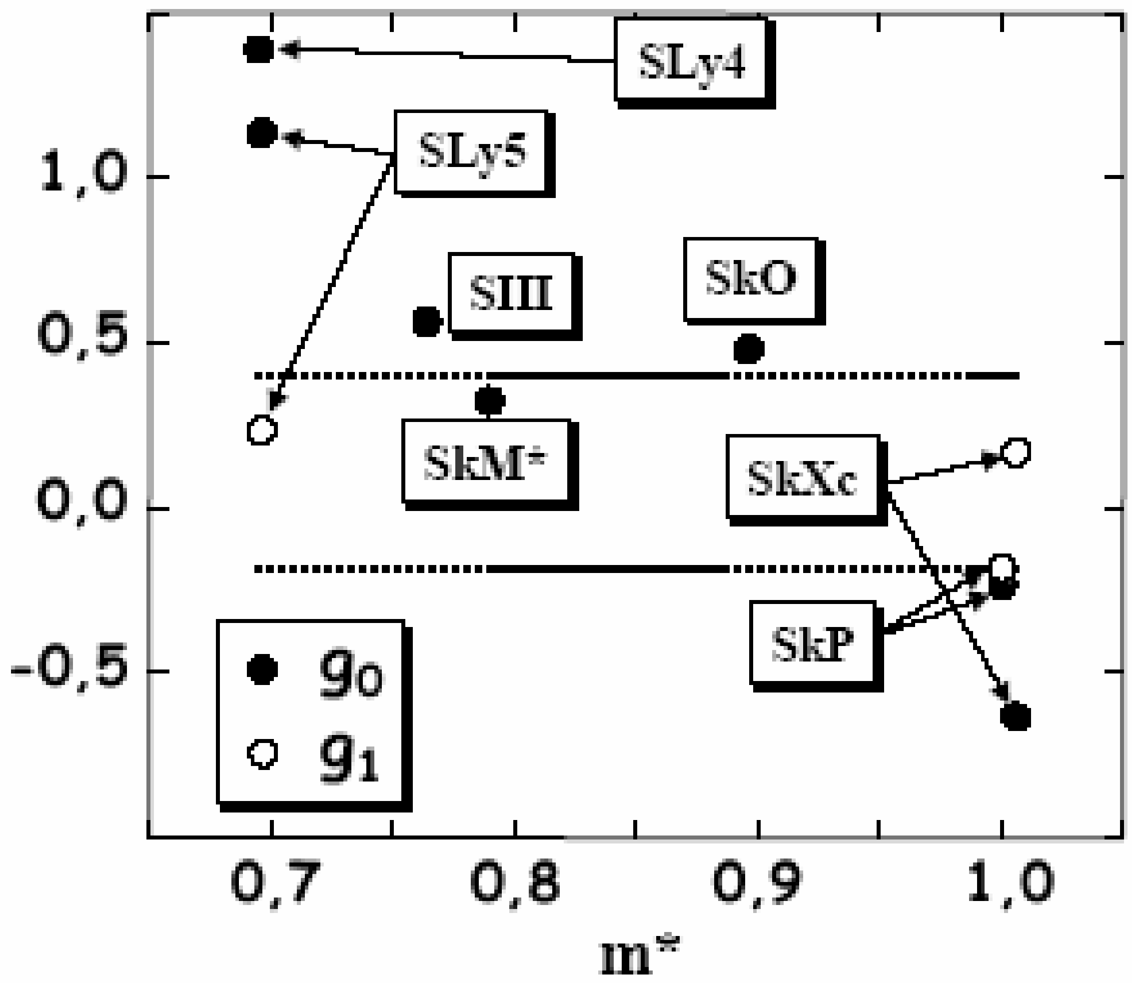
Landau parameters in Skyrme

$$g_0 = N_0(2C_0^s + 2C_0^T \beta \rho_0^{2/3}),$$

$$g_1 = -2N_0 C_0^T \beta \rho_0^{2/3},$$

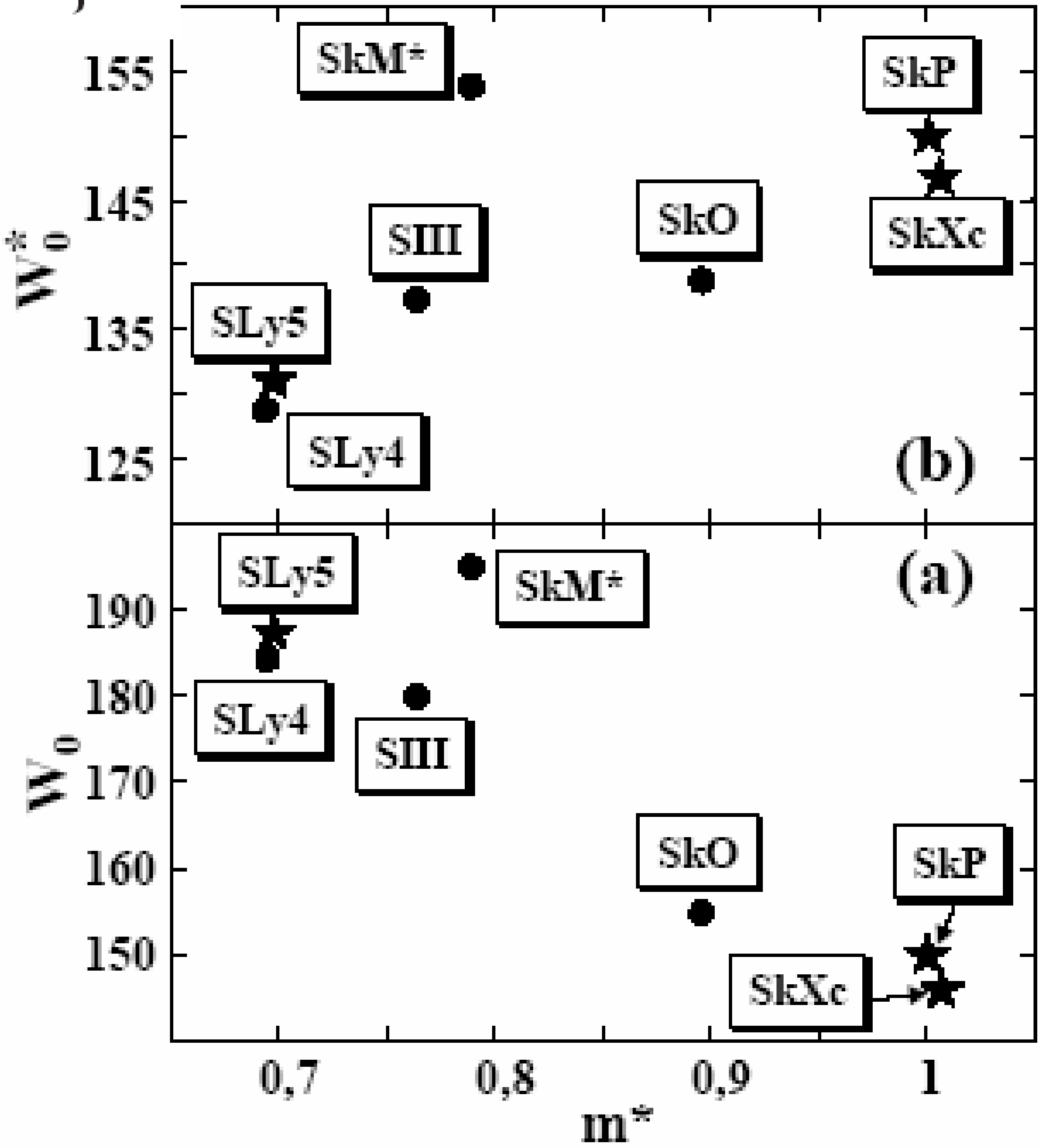
$$\beta = (3\pi^2/2)^{2/3},$$

$$N_0^{-1} = \pi^2 \hbar^2 / 2m^* k_F$$



$$V_{LS}(q,r) \approx \frac{m^\star(r)}{m}\left\{\frac{W_0}{r}\rho_0'(r) \pm \frac{W_1}{r}\rho_1'(r)\right\}\ell s.$$

$$C_t^{\nabla J} = -\frac{1}{2}W_t$$



Spin orbit strength

N=Z nuclei

N=Z nuclei not
well described!